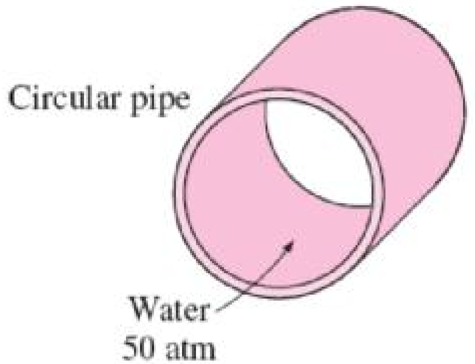
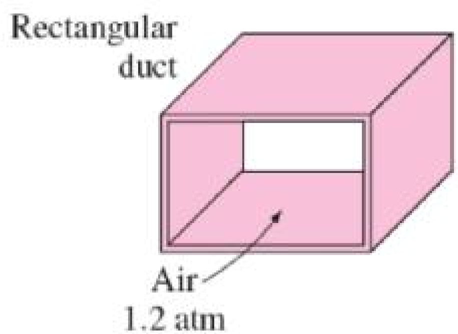
Flow Through Pipes

INTRODUCTION

- Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- We pay particular attention to *friction*, which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts.
- The pressure drop is then used to determine the *pumping power requirement*.





- Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

INTRODUCTION

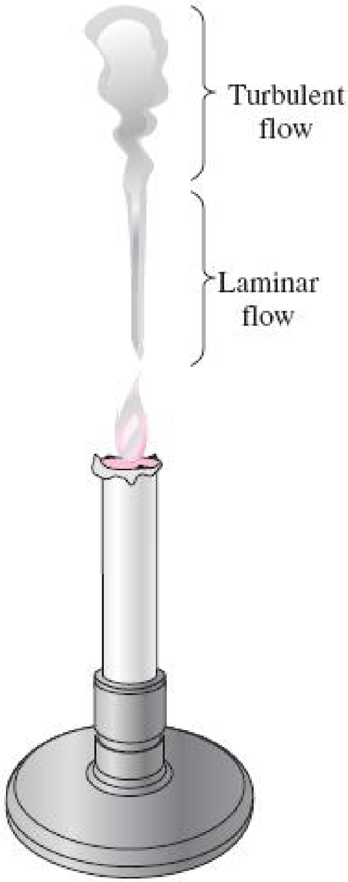
- Internal flows through pipes, elbows, tees, valves, etc., as in this oil refinery, are found in nearly every industry.

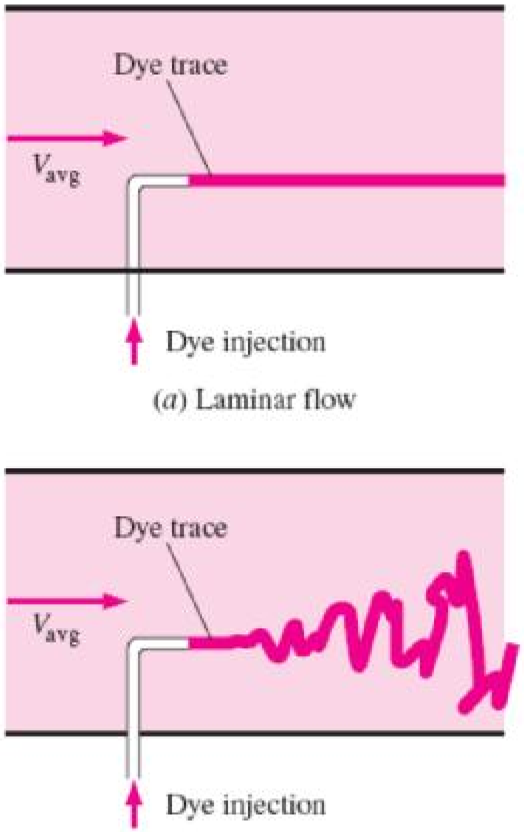
LAMINAR AND TURBULENT FLOWS

- Laminar: Smooth streamlines and highly ordered motion.
- Turbulent: Velocity fluctuations and highly disordered motion.
- Transition: The flow fluctuates between laminar and turbulent flows.

- Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.

- Most flows encountered in practice are turbulent.





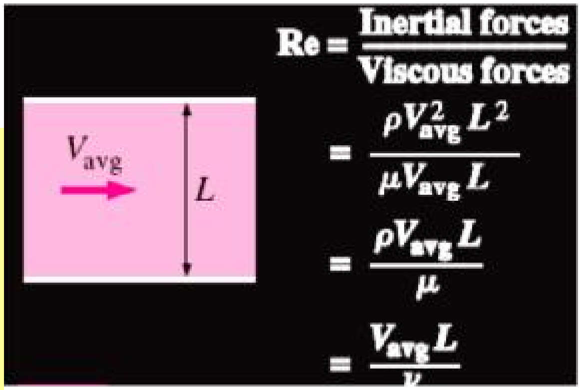
(a) Laminar flow

(b) Turbulent flow

Reynolds Number

- The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*.
- The flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* (Reynolds number).

$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg}D}{\nu} = \frac{\rho V_{avg} D}{\mu}$$



At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (turbulent).

At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line" (laminar).

Critical Reynolds number, Re_{cr}: The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

Reynolds Number

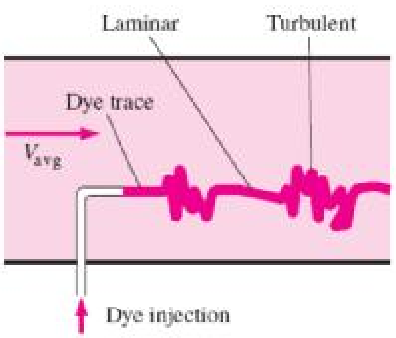
- For flow through noncircular pipes, the Reynolds number is based on the hydraulic diameter.

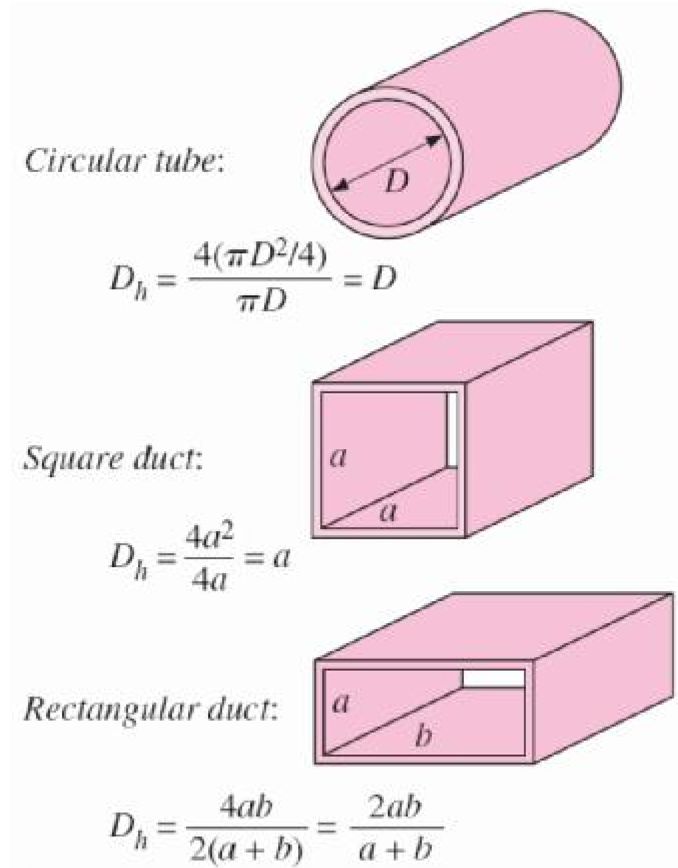
$$D_h = \frac{4A_c}{p}$$

- For flow in a circular pipe:

$$\begin{aligned} \text{Re} &\lesssim 2300 && \text{laminar flow} \\ 2300 \lesssim \text{Re} &\lesssim 4000 && \text{transitional flow} \\ \text{Re} &\gtrsim 4000 && \text{turbulent flow} \end{aligned}$$

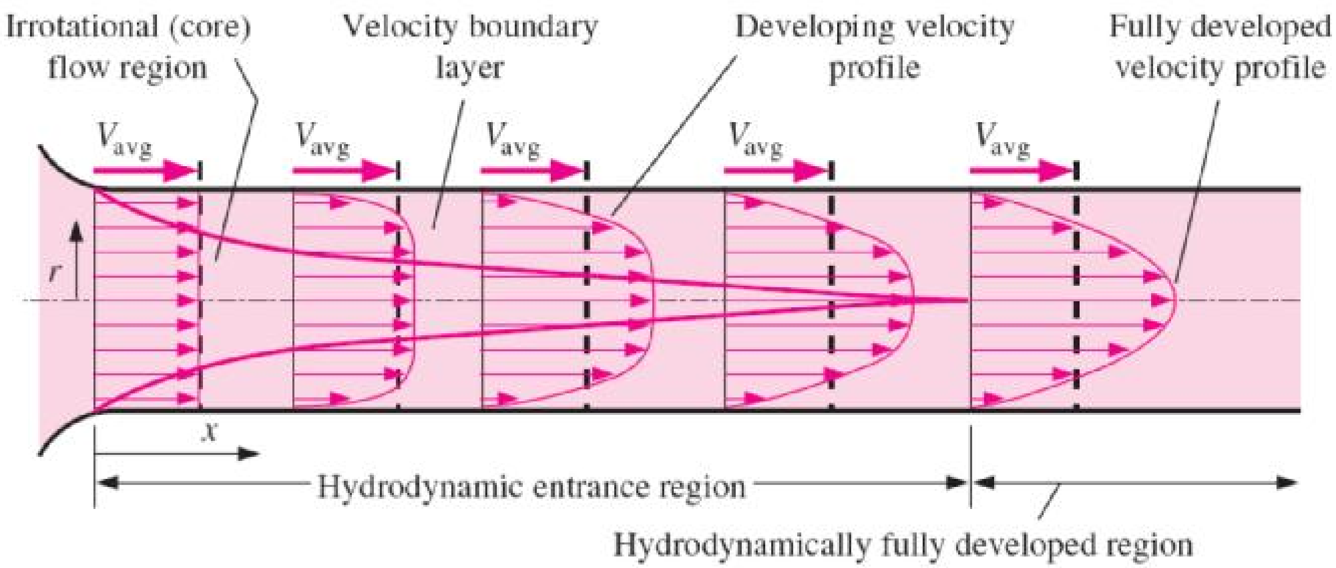
- In the transitional flow region of 2300 £ Re £ 4000, the flow switches between laminar and turbulent seemingly randomly.





THE ENTRANCE REGION

Velocity boundary layer: The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.
Boundary layer region: The viscous effects and the velocity changes are significant.
Irrotational (core) flow region: The frictional effects are negligible, and the velocity remains essentially constant in the radial direction.



The development of the velocity boundary layer in a pipe. The developed average velocity profile is parabolic in laminar flow, but somewhat flatter or fuller in turbulent flow.

THE ENTRANCE REGION

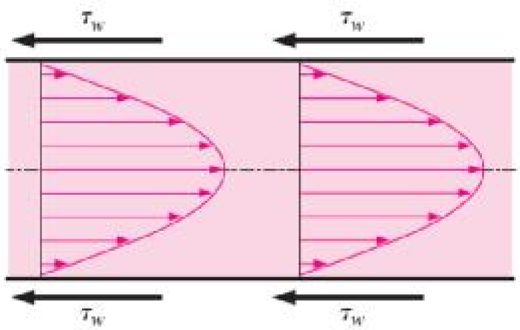
Hydrodynamic entrance region: The region from the pipe inlet to the point at which the boundary layer merges at the centerline.

Hydrodynamic entry length L_h: The length of this region.

Hydrodynamically developing flow: Flow in the entrance region. This is the region where the velocity profile develops.

Hydrodynamically fully developed region: The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

Fully developed: When both the velocity profile and the normalized temperature profile remain unchanged.

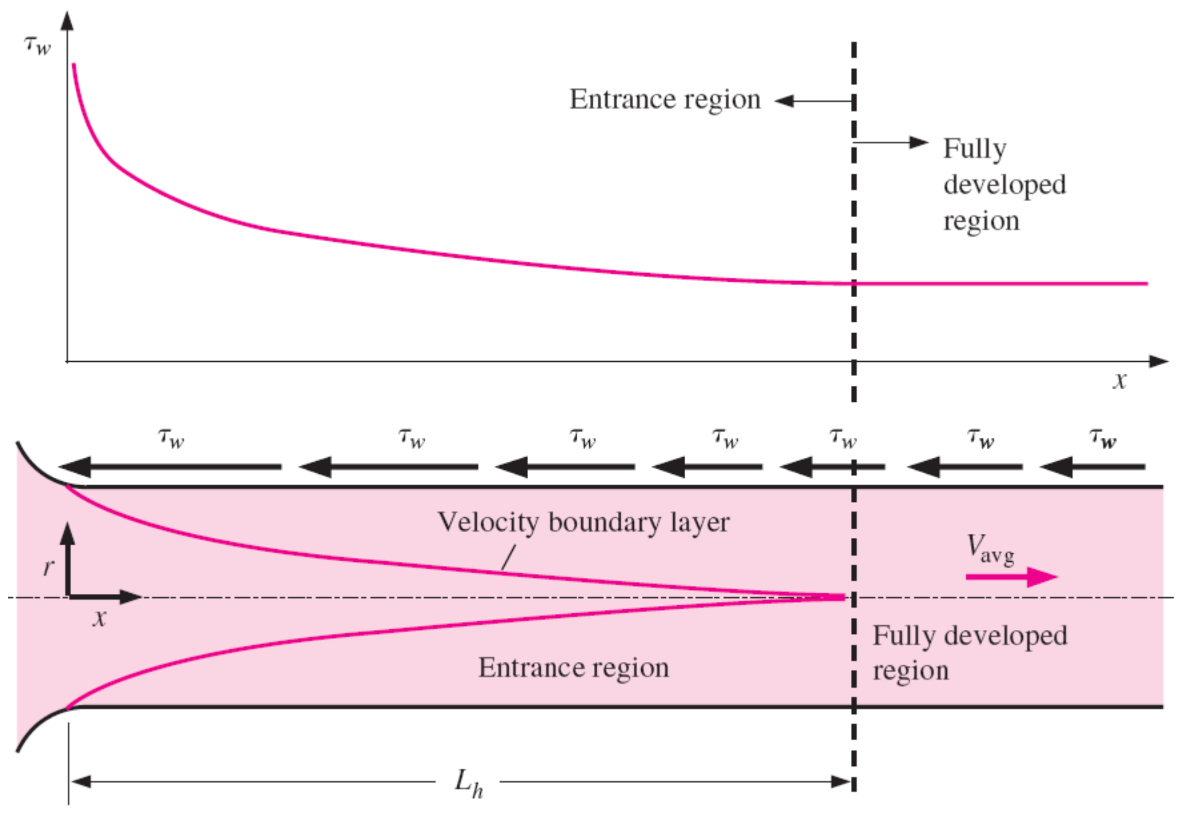


$$\frac{\partial u(r, x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$

In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.

THE ENTRANCE REGION

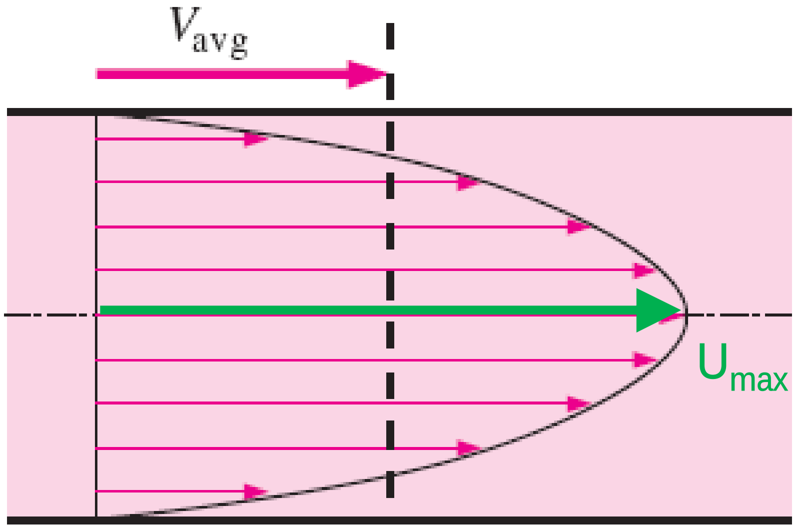
The pressure drop is *higher* in the entrance regions of a pipe, and the effect of the entrance region is always to *increase* the average friction factor for the entire pipe.



The average velocity

- Average velocity V_{avg} is defined as the average speed through a cross section. For fully developed laminar pipe flow, V_{avg} is half of the maximum velocity.

$$U_{max} = 2\, V_{avg}$$



Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.

$$\frac{L_{h,\,\text{laminar}}}{D} \cong 0.05\text{Re}$$

hydrodynamic entry length for laminar flow

$$\frac{L_{h,\,\text{turbulent}}}{D} = 1.359\text{Re}^{1/4}$$

hydrodynamic entry length for turbulent flow

$$\frac{L_{h,\,\text{turbulent}}}{D} \approx 10$$

hydrodynamic entry length for turbulent flow, an approximation

Pressure Drop and Head Loss

A quantity of interest in the analysis of pipe flow is the *pressure drop* ΔP since it is directly related to the power requirements of the fan or pump to maintain flow.

Laminar flow: $$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

In practice, it is found convenient to express the pressure loss for all types of fully developed internal flows (laminar or turbulent flows, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes) as

Pressure loss: $$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

where $\rho V_{avg}^2/2$ is the *dynamic pressure* and f is the **Darcy friction factor,**

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

It is also called the Darcy–Weisbach friction factor,

Pressure Drop and Head Loss

At laminar flow, The friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface as shown *in the following eq.*

$$\textit{Circular pipe, laminar:} \qquad f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\text{Re}}$$

In the analysis of piping systems, pressure losses are commonly expressed in terms of the *equivalent fluid column height, called the* head loss *h*. Noting from fluid statics that P = r gh *and thus a pressure difference of P* corresponds to a fluid height of $h = P/$r g, *the pipe head loss is obtained* by dividing ΔP_L by r g *to give*

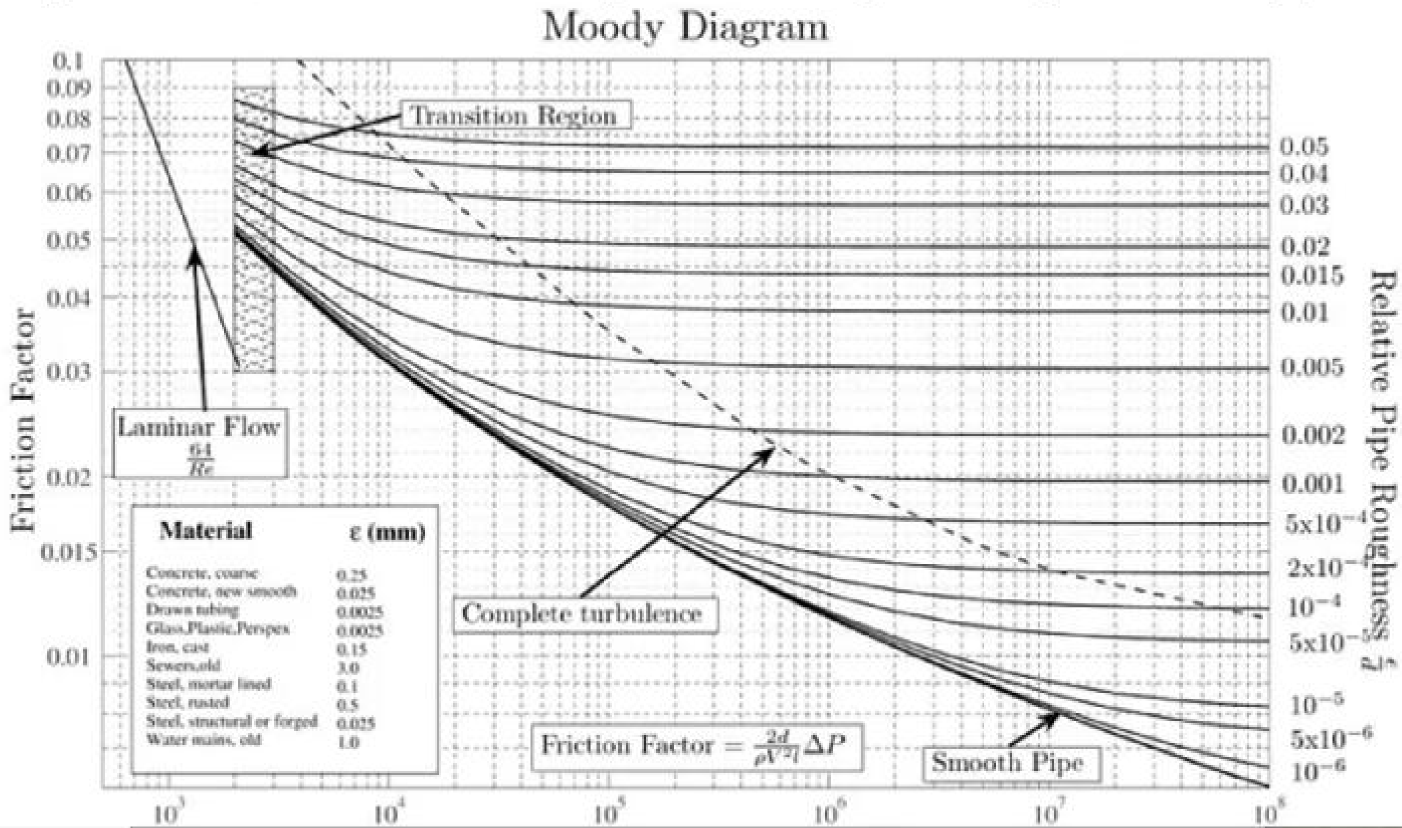
$$\textit{Head loss:} \qquad h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

Once the pressure loss (or head loss) is known, the required pumping power *to overcome the pressure loss is determined from*

$$\dot{W}_{pump,L} = \dot{Q}\Delta P_L = \dot{Q}\rho g h_L = \dot{m} g h_L$$

The Moody Chart

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the relative roughness **e**/*D*, ***which is the ratio of the*** mean height of roughness of the pipe to the pipe diameter.

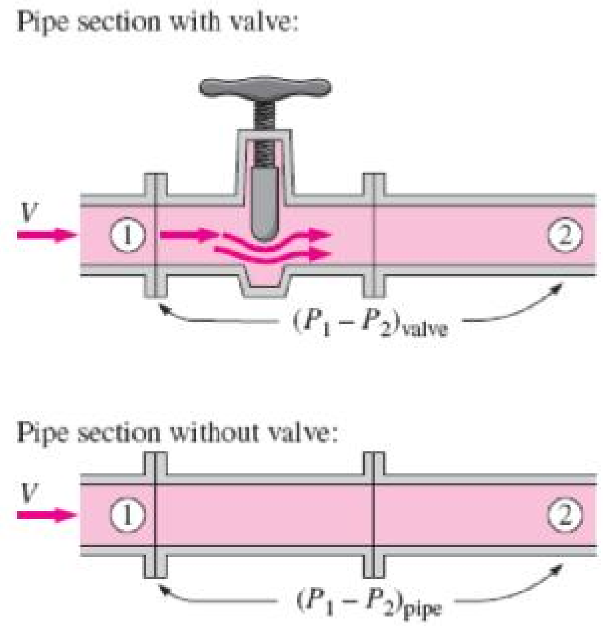


MINOR LOSSES

- The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, expansions, and contractions in addition to the pipes.
- These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce.
- In a typical system with long pipes, these losses are minor compared to the total head loss in the pipes (the major losses) and are called minor losses.
- Minor losses are usually expressed in terms of the loss coefficient K_L.

$$K_L = \frac{h_L}{V^2/(2g)}$$

$h_L = \Delta P_L/\rho g$ Head loss due to component



$$\Delta P_L = (P_1 - P_2)_{valve} - (P_1 - P_2)_{pipe}$$

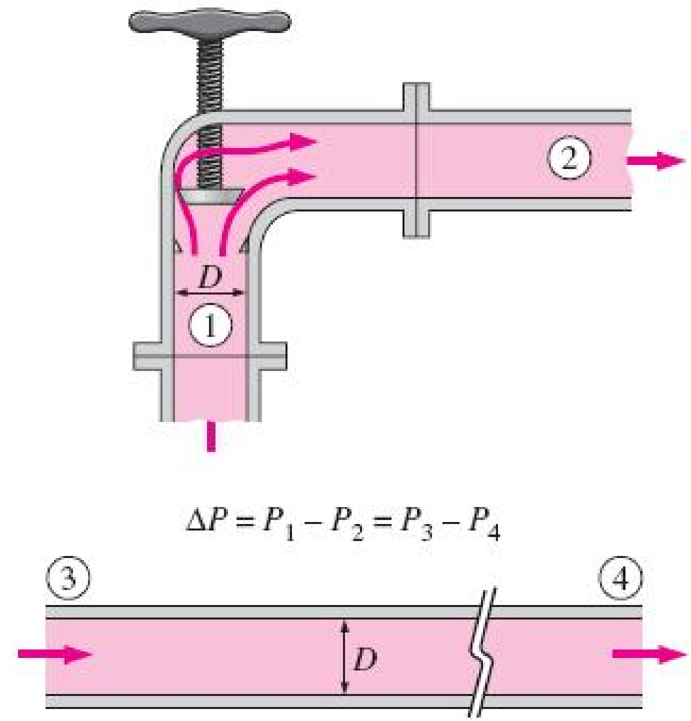
MINOR LOSSES

When the inlet diameter equals outlet diameter, the loss coefficient of a component can also be determined by measuring the pressure loss across the component and dividing it by the dynamic pressure:

$$K_L = \Delta P_L / (\rho V^2/2).$$

When the loss coefficient for a component is available, the head loss for that component is

$$h_L = K_L \frac{V^2}{2g}$$ *Minor loss*



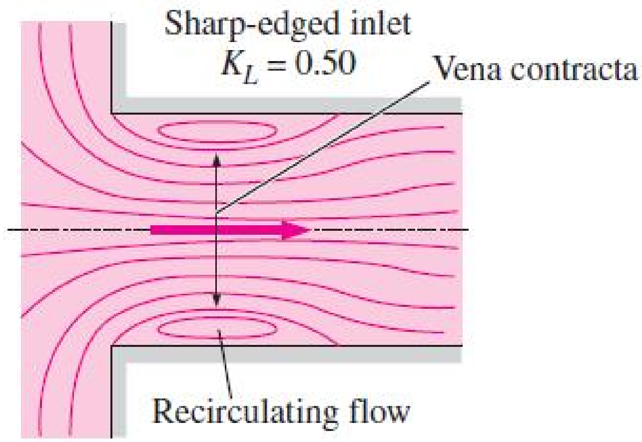
Total head loss

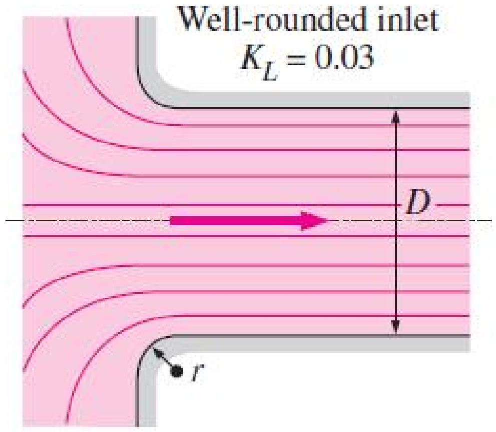
$$h_{L,\,\text{total}} = h_{L,\,\text{major}} + h_{L,\,\text{minor}}$$

$$= \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,j}\frac{V_j^2}{2g}$$

Total head loss (D = constant)

$$h_{L,\,\text{total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$



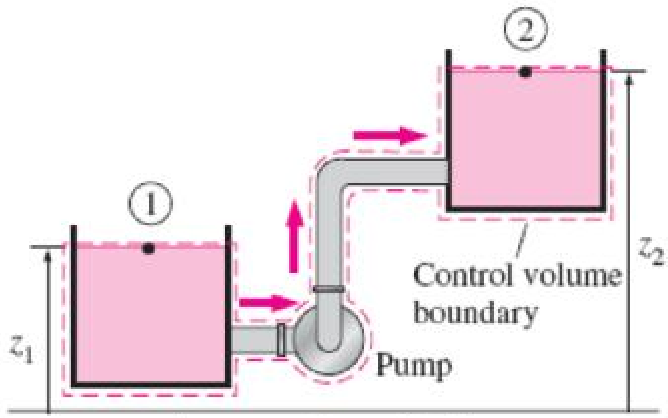


Piping Systems with Pumps and Turbines

When a piping system involves a pump and/or turbine, the steady-flow energy equation on a unit-mass basis can be expressed as

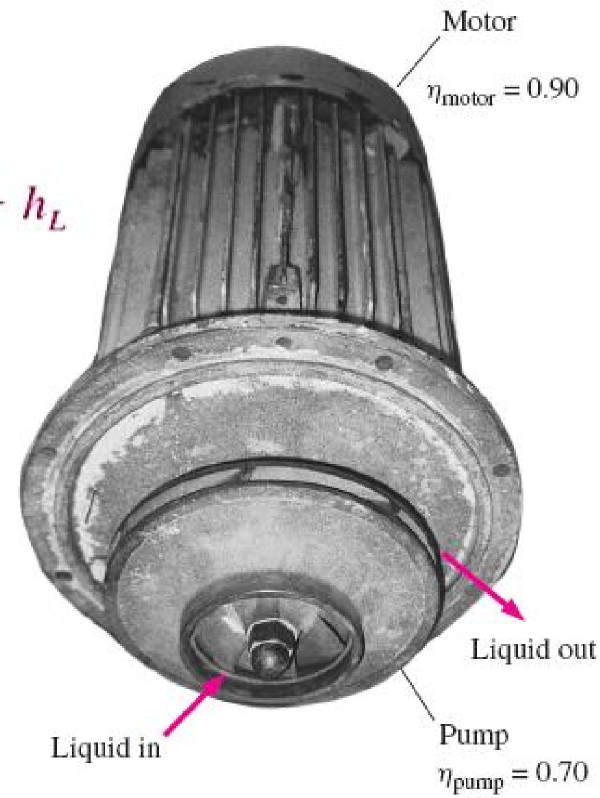
$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{pump,\,u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{turbine,\,e} + h_L$$

$$\dot{W}_{pump} = \frac{\rho Q g h_{pump}}{\eta_{pump}}$$



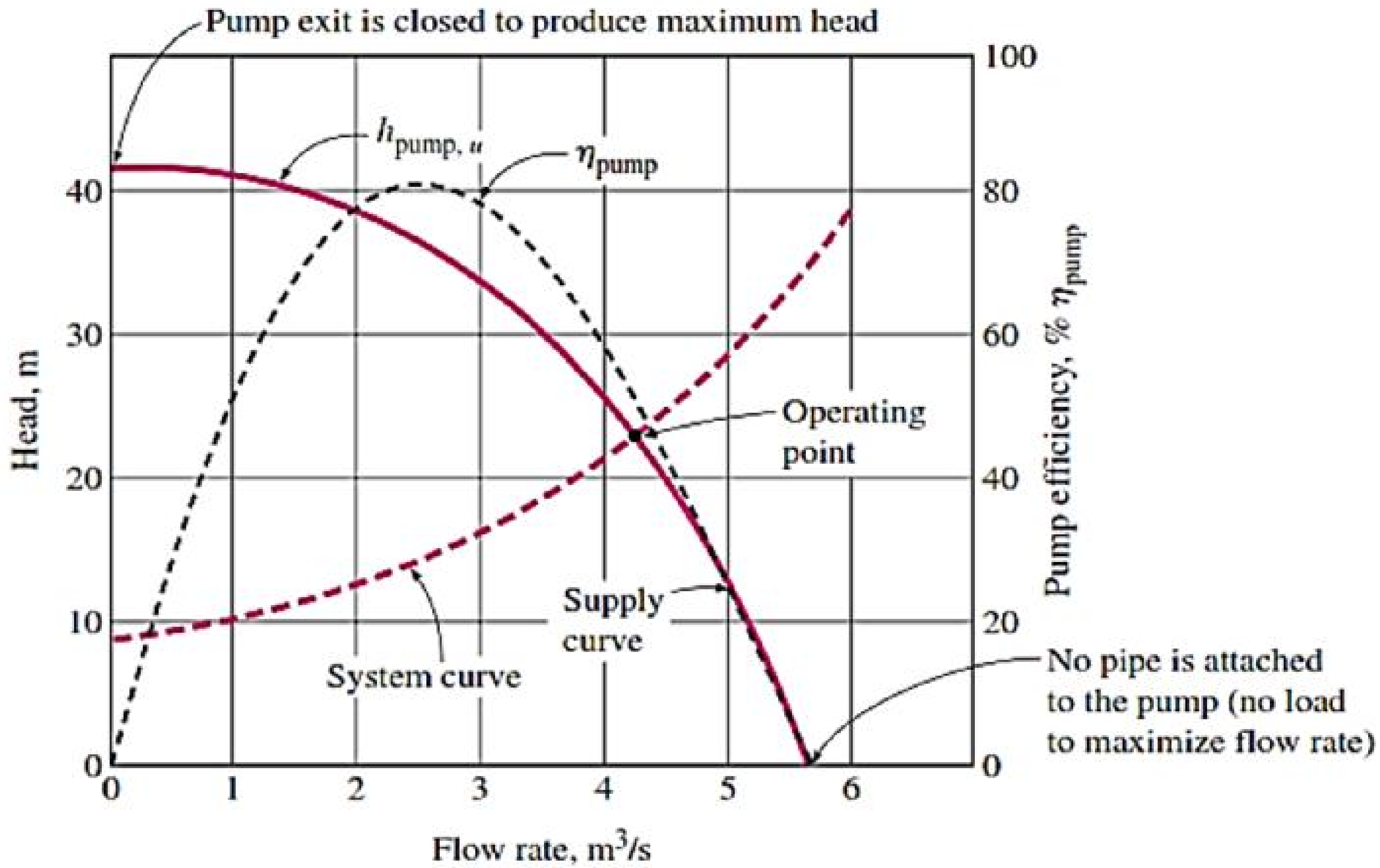
$h_{pump,\,u} = (z_2 - z_1) + h_L$

$\dot{W}_{pump,\,u} = \rho \dot{V} g h_{pump,\,u}$



$\eta_{pump\text{–}motor} = \eta_{pump}\eta_{motor}$
$= 0.70 \times 0.90 = 0.63$

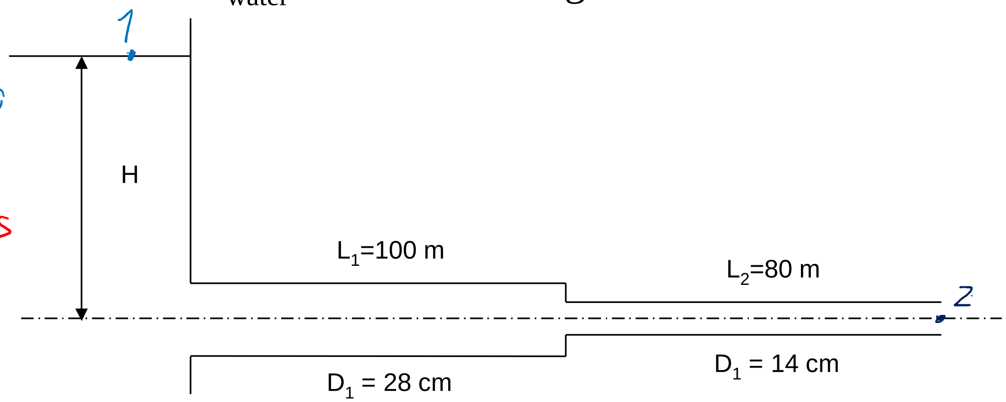
Pump Operating Point



Characteristic pump curves for centrifugal pumps, the system curve for a piping system, and the operating point.

Examples:

1. Find the head H in the shown tank required to derive water flow of 0.314 m^3/s through the two pipes shown in series. The pipes were made of cast iron. $\nu_{water} = 10^{-6}\ m^2/s$ – Neglect minor losses.



- Solution:

Energy eq. should be applied between ① & ②

$$\frac{P_1}{\delta g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\delta g} + \frac{V_2^2}{2g} + Z_2 + h_{losses}$$

(with $\frac{P_1}{\delta g}$, $\frac{V_1^2}{2g}$, $\frac{P_2}{\delta g}$ and Z_2 struck out; $\frac{V_1^2}{2g} = 0$, $Z_2 = 0$)

$$\therefore H = h_{losses} + \frac{V_2^2}{2g}$$

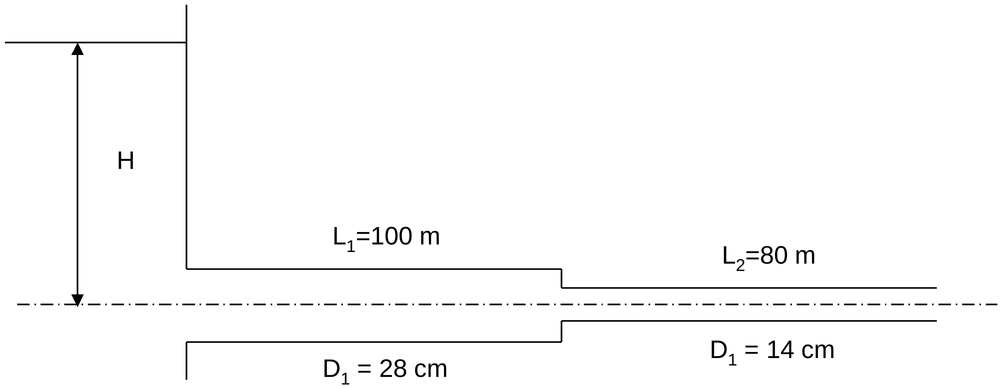
$$\because V_2 = \frac{Q}{\frac{\pi}{4} D_2^2} = \frac{0.314}{\frac{\pi}{4}(0.14)^2} = 20.39\ m/s$$

$$h_{losses} = h_{major\ losses} = F \frac{L}{D} \frac{V^2}{2g}$$

$$h_{losses} = h_{L_1} + h_{L_2} \Rightarrow h_L = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

Examples:

- Solution continued



$$V_1 = \frac{Q}{\frac{\pi}{4} D_1^2} = \frac{0.314}{\frac{\pi}{4}(0.28)^2} = 5.1 \text{ m/sec}$$

$$\therefore h_1 = F_1 \frac{100}{0.28} \frac{(5.1)^2}{2 * 9.81} + F_2 \frac{80}{0.14} \frac{(20.39)^2}{2 * 9.81}$$

We have to calculate Re_1 & ϵ/D_1 to get F_1

$$Re_1 = \frac{V_1 D_1}{\nu} = \frac{5.1 * 0.28}{10^{-6}} = 1.428 * 10^6 \text{ (turbulent flow)}, \quad \epsilon/D_1 = \frac{0.26}{280} = 0.000928$$

$$Re_2 = \frac{V_2 D_2}{\nu} = \frac{20.39 * 0.14}{10^{-6}} = 2.854 * 10^6, \quad \epsilon/D_2 = \frac{0.26}{140} = 0.001857$$

From Moody diagram $F_1 = 0.0205$ $F_2 = 0.024$

$$\therefore h_{losses} = 0.0205 \frac{100}{0.28} \frac{(5.1)^2}{2 * 9.81} + 0.024 \frac{80}{0.14} \frac{(20.39)^2}{2 * 9.81} = 300.3 \text{ m}$$

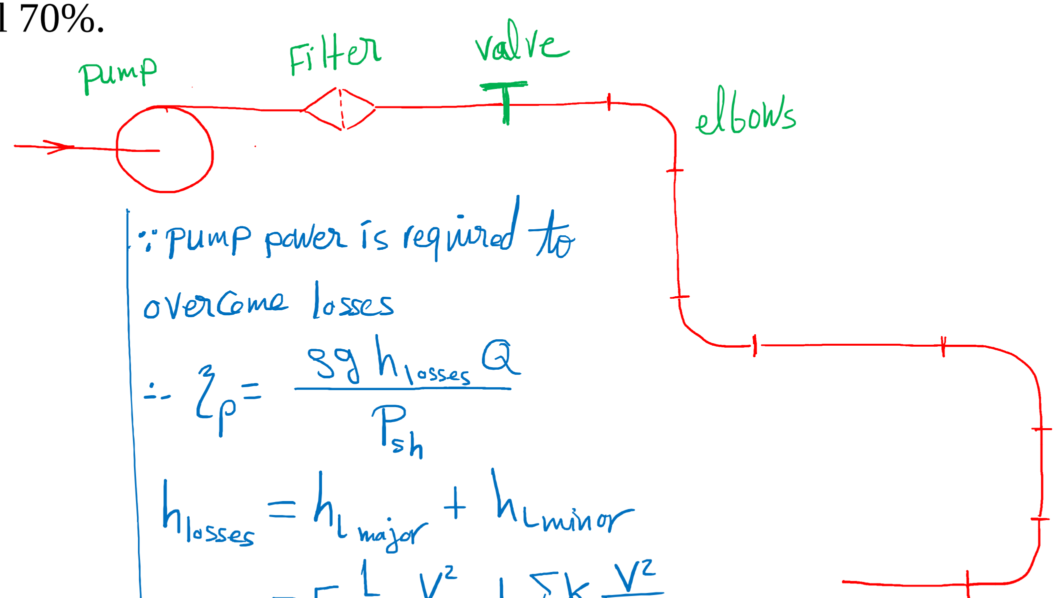
$$\therefore H = 300.3 + \frac{(20.39)^2}{2 * 9.81} \Rightarrow \boxed{H = 321.5 \text{m}}$$

Examples:

2. Determine the power of the pump required to overcome the head losses through a pipe has 1 km length and 30 cm diameter to derive water flow of 0.3 m^3/s. The pipe has four elbows, one filter, and one valve.

$K_{elbow} = 0.7$, $k_{filter} = 7$, $k_{valve} = 5$, $\varepsilon = 0.1$ mm, $\nu_{water} = 10^{-6}\ m^2/s$.

Take: the efficiency of the pump equal 70%.



- Solution:

$L = 1000\ m$, $D = 30\ cm$, $Q = 0.3\ m^3/sec$

$K_{elbows} = 0.7 * 4$, $\epsilon = 0.1\ mm$

$K_{Filter} = 7 * 1$, $\nu_{water} = 10^{-6}\ m^2/sec$

$K_{valve} = 5 * 1$, $\eta_p = 70\% = 0.7$

Required:-

P_{sh}

∵ pump power is required to overcome losses

$$\therefore \eta_p = \frac{\rho g\, h_{losses}\, Q}{P_{sh}}$$

$$h_{losses} = h_{L\,major} + h_{L\,minor}$$

$$= F \frac{L}{D} \frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

Examples:

- Solution continued

$$h_L = \left(f\frac{L}{D} + \sum K\right) V^2/2g$$

$$V = \frac{Q}{\frac{\pi}{4}D^2} \Rightarrow V = \frac{4 * 0.3}{\pi * (0.3)^2} = 4.25\ m/s$$

$$Re_D = \frac{VD}{\nu} = \frac{4.25 * 0.3}{10^{-6}} = 1.27 * 10^6 > 4000$$

turbulent

$$\epsilon/D = \frac{0.1}{300} = 0.00034$$

From Moody diagram get $F = 0.017$

$$\sum K = 4*0.7 + 7 + 5 = 14.8$$

$$\therefore h_L = \left(0.017 * \frac{1000}{0.3} + 14.8\right) * \frac{(4.25)^2}{2 * 9.81}$$

$$\therefore h_L = 65.8\ m$$

$$\therefore 0.7 = \frac{1000 * 9.81 * 65.8 * 0.3}{P_{sh}}$$

$$P_{sh} = 276614.3\ W = 276.61\ kW$$

Examples:

3. Design a pipe that required to derive water flow of 0.28 m^3/s over 1200 m length to overcome a head losses of 10 m. take $\nu_{water} = 10^{-6}$ m^2/s and $\varepsilon = 0.1$ mm.

- Solution:

Design of the pipe (D)

① Assume suitable value for F_1

② Calculate D_1 from $h_L = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g}$

③ from D_1 calculate $Re_1 = \frac{V_1 D_1}{\nu}$

④ From Re_1 & ϵ / D_1 get new f_2 from moody diagram

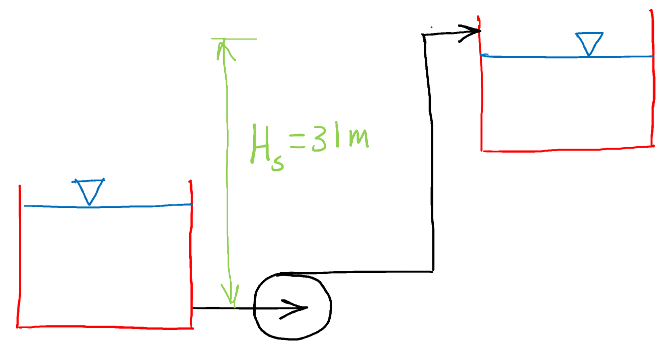
⑤ Recalculate D_2 from $h_L = f_2 \frac{L}{D_2} \frac{V_2}{2g}$

Check the relative error at each trial

$$\frac{D_2 - D_1}{D_1} = \text{Relative Error}$$

Examples:

4. An oil (S=0.82) is pumped between 2 storage tanks in a pipe with the following characteristics L=2440 m, D=20 cm, f=0.02 and Σk=12.5. Oil level in upper tank is 31 m above that in lower tank. Use the following pump data evaluate the oil discharge and the power required for the pump.
   - Q l/s 0 15 30 45 60 75 100
   - H_p m 55 54 53 52 49 44 35
   - η 0 0.4 0.6 0.7 0.75 0.7 0.5



- Solution:

Q (l/s)	0	15	30	45	60	75	100
Q(m^3/s)	0	0.015	0.03	0.045	0.06	0.075	0.1
H_p (m)	55	54	53	52	49	44	35
η	0	0.4	0.6	0.7	0.75	0.7	0.5

$$H_{system} = H_s + \left(f\frac{L}{D} + \Sigma k\right)\frac{V^2}{2g}$$

since $V = \dfrac{Q}{\frac{\pi}{4}D^2}$

The required Q is $Q_{o.p}$ (flow at the operating point)

Examples:

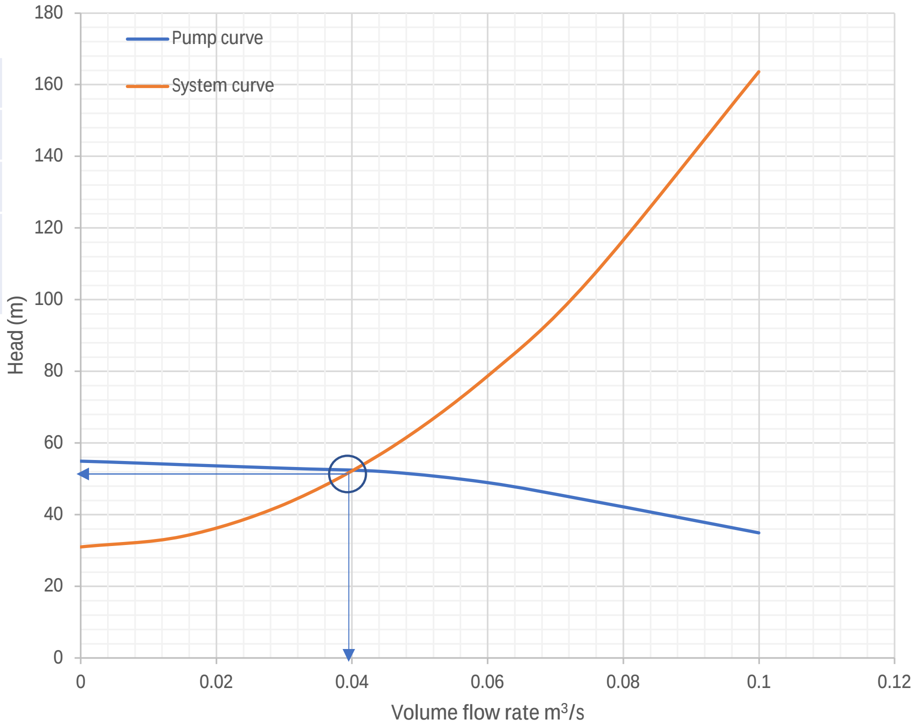
- Solution continued:

Q (l/s)	0	15	30	45	60	75	100
$Q(m^3/s)$	0	0.015	0.03	0.045	0.06	0.075	0.1
V(m/s)	0	0.477	0.955	1.433	1.910	2.388	3.184
H_{system} (m)	31	33.9834	42.9336	57.8506	78.7344	105.585	163.596

From the graph get $Q_{o.p} = 0.04\ m^3/s$, $H_{o.p} = 52m$

since $\eta_{o.p} = \dfrac{\gamma g Q_{o.p}\ H_{o.p}}{P_{sh}}$

Plot graph between η and Q



Examples:

- Solution continued:

From curve get $\eta_{o.p} = 67\%$

$$\therefore \; 0.67 = \frac{1000 * 9.81 * 0.04 * 52}{P_{sh}}$$

$$P_{sh} = 30454.9 \; W$$

$$\boxed{P_{sh} = 30.45 \; kW}$$